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Isospin symmetry breaking in $\rho \to \pi\gamma$ decay

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Abstract. In terms of effective field theory and mixed-propagator approach, we show that there is a larger hidden effect of isospin breaking in $\rho^0 \to \pi^0\gamma$ decay due to a ω exchange, $\rho^0 \to \omega \to \pi^0\gamma$. The branching ratio is predicted as $B(\rho^0 \to \pi^0\gamma) = (11.67 \pm 2.0) \times 10^{-4}$, which is much larger than Particle Data Group's datum $(6.8 \pm 1.7) \times 10^{-4}$ and one of charged mode, $B(\rho^\pm \to \pi^\pm\gamma) = (4.5 \pm 0.5) \times 10^{-4}$.

PACS. 14.40.Cs Other mesons with $S = C = 0$, mass < 2.5 GeV – 13.25.Jx Decays of other mesons – 12.40.Vv Vector-meson dominance – 13.40.Hq Electromagnetic decays

1 Introduction

In hadron physics, the isospin symmetry or charge symmetry [1] is broken by inequality of the light-quark masses, especially $m_u \neq m_d$, and electromagnetic interaction of hadrons. This breaking of the isospin symmetry induces various measurable physics effects such as π^0-η, Λ-Σ^0 mixing, $\omega \to \pi^+\pi^-$ decay, etc. It is common knowledge that the effects of isospin breaking can be omitted in the isospin conservation processes. However, this argument is not always right. The purpose of this paper is to show that a large-isospin breaking effect is in isospin conservation decay $\rho^0 \to \pi^0\gamma$ (so-called hidden isospin symmetry breaking effect). Thus, the real branching ratio for this decay should be much larger than the datum cited by PDG-2000 [2] and charged mode.

The anomalous-like radiative decays of light flavour vector mesons have been observed by several group: The branching ratio for charged ρ is $B(\rho^\pm \to \pi^\pm\gamma) = (4.5 \pm 0.5) \times 10^{-4}$ [3]. The branching ratios for ρ^0 and ω decays were first obtained in refs. [4–6] by using the data of e^+e^- collider in neutral detector, but the triangle anomaly contribution of QCD was ignored in their analysis. Benayoun *et al.* have reanalyzed the ρ^0 and ω decay to a pseudoscalar meson plus a photon via taking into account the triangle anomaly contribution [7]. The authors used two models (M_1 and M_2) [8,9] to fit the data. It is a bit surprising that they found two local minima for both the two model fits, one with $\chi^2/N_{\rm df} \simeq 0.5$ (solution A) and another one with $\chi^2/N_{\rm df} \simeq 0.7$ (solution B). See table 1, where the phase Φ_V of vector mesons is defined via the $e^+e^- \to \pi^0\gamma$ cross-section (eq. (1) of ref. [7]). Although the solution A has smaller $\chi^2/N_{\rm df}$, and the phase difference $\Phi_\phi - \Phi_\rho$ is found around 210° in solution A, close to expectation from the quark model (180°), the ρ^0 branching ratio is much larger than expected from the charged mode or from $SU(3)$ symmetry. Thus, the authors took solution B as final result to be in agreement with isospin symmetry. However, this conclusion is incorrect. In this paper we shall see what happens there.

The paper is organized as follows: In sect. 2, we first give the formula of the transition amplitude for ρ, $\omega \to \pi\gamma$. Then we will prove this formula via two independent methods: the effective field theory approach and the mixed-propagator approach. In sect. 3, we will provide final numeric results, and a brief conclusion.

2 Transition amplitude for $\mathrm{V} \to \pi\gamma$

The transition matrix element for vector mesons decay to a pion and a photon is

$$\langle \pi(k)\gamma(q_1)|V(q_2)\rangle = i f_{V\pi\gamma}\epsilon^{\mu\nu\alpha\beta} q_{1\mu}\epsilon^*_\nu q_{2\alpha} e_\beta, \tag{1}$$

where e_μ and ϵ_μ are the polarized vectors of vector mesons and photon, respectively. In the large-N_c limit and in the chiral limit, if we ignore possible contribution from resonance exchange, the exact isospin symmetry implies

$$f_{\rho^\pm\pi^\pm\gamma} = f^{(0)}_{\rho^0\pi^0\gamma} = \frac{1}{3} f^{(0)}_{\omega\pi^0\gamma}, \tag{2}$$

where the superscript "(0)" denotes absence of the resonance exchange. A complete consideration for ρ^0 and ω^0

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Table 1. Branching ratios and phases obtained from the fit to the $e^+e^- \to \pi^0\gamma$ cross-section using two ρ^0 models and assuming $\Phi_\rho = \Phi_\omega$. The table is abstracted from table 4 of ref. [7].

$\Gamma(\pi^0 \to \gamma\gamma) = 7.7$ eV	Model M_1		Model M_2	
	Solution A	Solution B	Solution A	Solution B
$\rho^0 \to \pi^0\gamma$ (units 10^{-4})	(11.51 ± 2.00)	(6.17 ± 1.57)	(11.67 ± 2.00)	(6.77 ± 1.72)
$\omega \to \pi^0\gamma$ (units 10^{-2})	(8.62 ± 0.26)	(8.39 ± 0.24)	(8.64 ± 0.27)	(8.39 ± 0.24)
$\phi \to \pi^0\gamma$ (units 10^{-3})	(1.15 ± 0.13)	(1.20 ± 0.16)	(1.21 ± 0.16)	(1.26 ± 0.17)
$\Phi_\rho = \Phi_\omega$ (degrees)	-94 ± 8	124 ± 9	-90 ± 7	125 ± 11
Φ_ϕ (degrees)	114 ± 14	248 ± 9	119^{+11}_{-18}	248^{+18}_{-10}
χ^2/dof	29/57	38/57	29/57	36/57

decays should include the contribution from the following resonance exchange:

$$
f^{(c)}_{\rho^0\pi^0\gamma} = f^{(0)}_{\rho^0\pi^0\gamma} + \frac{\Pi_{\rho\omega}(m_\rho^2) f^{(0)}_{\omega\pi^0\gamma}}{m_\rho^2 - m_\omega^2 + i m_\omega \Gamma_\omega},
$$

$$
f^{(c)}_{\omega\pi^0\gamma} = f^{(0)}_{\omega\pi^0\gamma} + \frac{\Pi_{\rho\omega}(m_\omega^2) f^{(0)}_{\rho^0\pi^0\gamma}}{m_\omega^2 - m_\rho^2 + i m_\rho \Gamma_\rho}, \tag{3}
$$

where the momentum-dependent ρ^0-ω mixing amplitude $\Pi_{\rho\omega}(q^2)$ is defined in the following effective Lagrangian (eq. (5)). The relations in eq. (3) hold more generally. In other words, the couplings $f^{(0)}_{\rho^0\pi^0\gamma}$ and $f^{(0)}_{\omega\pi^0\gamma}$ can include corrections beyond the large-N_c limit and the chiral limit. Meanwhile, relation (2) will be broken, but this breaking is slight. This point can be checked by using the datum of $\rho^\pm \to \pi^\pm\gamma$ decay and $\omega \to \pi^0\gamma$ decay if we believe that $f_{\rho^\pm\pi^\pm\gamma} = f^{(0)}_{\rho^0\pi^0\gamma}$.

It must be pointed out that eq. (3) is a non-perurbative result instead of a perturbative expression. In the rest of this section we will prove it by two independent methods: effective field theory approach and mixed-propagator approach, respectively.

2.1 Effective field theory approach

The most general effective Lagrangian concerning $\rho^0 \to \pi^0\gamma$ and $\omega \to \pi^0\gamma$ decays is given as follows:

$$
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{I}} + \mathcal{L}_2 + \mathcal{L}_{\mathrm{ChPT}} + \mathcal{L}_{\mathrm{WZW}}, \tag{4}
$$

where $\mathcal{L}_{\mathrm{ChPT}}$ is the Lagrangian of the chiral perturbative theory, $\mathcal{L}_{\mathrm{WZW}}$ is the Wess-Zumino-Witten Lagrangian, $\mathcal{L}_0$ and $\mathcal{L}_{\mathrm{I}}$ are free fields and interaction Lagrangians for vector mesons, respectively, $\mathcal{L}_2$ is the counterterm Lagrangian for $\mathcal{L}_0$ and $\mathcal{L}_{\mathrm{I}}$[1]. Explicitly, $\mathcal{L}_0$ and $\mathcal{L}_{\mathrm{I}}$ can be written as follows:

$$
\begin{aligned}
\mathcal{L}_0 = &-\frac{1}{4}(\partial_\mu \rho^i_\nu - \partial_\nu \rho^i_\mu)(\partial^\mu \rho^{i\nu} - \partial^\nu \rho^{i\mu}) \\
&-\frac{1}{4}(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)(\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) \\
&+\frac{1}{2}\tilde{m}_\rho^2 \rho^i_\mu \rho^{i\mu} + \frac{1}{2}\tilde{m}_\omega^2 \omega_\mu \omega^\mu + \cdots, \\
\mathcal{L}_{\mathrm{I}} = &\int \frac{\mathrm{d}^4 q}{(2\pi)^4} e^{iq\cdot x} \Big\{ \Pi_{\rho\omega}(q^2)\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)\omega^\mu(x)\rho^{0\nu}(q) \\
&+ i\epsilon^{ijk} f_{\rho\pi\pi}(q^2)\rho^\mu_i(q)\pi_j(x)\partial_\mu \pi_k(x) \\
&+ \epsilon^{\mu\nu\alpha\beta}\epsilon^{ijk} f_{\omega 3\pi}(q^2)\omega_\mu(q)\partial_\nu \pi_i(x)\partial_\alpha \pi_j(x)\partial_\beta \pi_k(x) \\
&+ \epsilon^{\mu\nu\alpha\beta} f_{\rho\pi\gamma}(q^2) q_\mu \rho^i_\nu(q)\partial_\alpha A_\beta(x)\pi_i(x) \\
&+ \epsilon^{\mu\nu\alpha\beta} f_{\omega\pi\gamma}(q^2) q_\mu \omega_\nu(q)\partial_\alpha A_\beta(x)\pi^0(x) + \cdots \Big\}.
\end{aligned} \tag{5}
$$

Several remarks are related to Lagrangian (5):

1. Focusing on any low-energy effective theory, a general knowledge is that any coupling should be momentum-dependent [10]. In Lagrangian (5) this statement has been exhibited via the momentum-dependence of the form factors $f_{\rho\pi\pi}(q^2)$, $f_{\omega 3\pi}(q^2)$, etc. All these form factors should be real function of the vector meson four-momentum squared, q^2. So that the unitarity of S-matrix prevents us from taking the complex mass square in the on-shell transition amplitude.
2. $\tilde{m}_\rho$ and $\tilde{m}_\omega$ in $\mathcal{L}_0$ are not the physical masses of ρ^0 and ω, since they will be shifted due to ρ^0-ω mixing[2]. The physical masses of ρ^0 and ω correspond to poles in their type-I complete propagators (fig. 1)[3], in which we have considered the contribution from the ρ^0-ω mixing

[1] An effective theory is renormalized if the number of coupling constants in the effective Lagrangian is infinite, *e.g.*, chiral perturbative theory. Here, we focus on the most general effective Lagrangian, thus it can be treated as a renormalized one.

[2] Of course, there should be other possible mechanisms which can also cause mass splitting of ρ^0 and ω. In this paper, however, we only provide a heuristical discussion. Thus, we ignore other ingredients here.

[3] Due to the effective Lagrangian (5), the complete propagators of ρ^0 and ω receive two kinds of contribution. One is from ρ^0-ω mixing and the other is from meson loops. We label them as type-I and type-II contribution, respectively. In addition, the renormalization does not shift the pole of the propagator, thus the pole in type-I complete propagators is the same as the one in complete propagators considering both kinds of contributions.

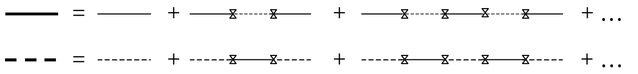

Fig. 1. Non-perturbative correction of ρ^0-ω mixing to the propagators of ρ^0 and ω. The physical masses of ρ^0 and ω correspond to the poles of type-I complete propagators. Here solid lines and dashed lines denote the non-physical propagator of ρ^0 and ω, respectively, and thick solid line and thick dashed line denote type-I complete propagators of ρ^0 and ω, respectively.

completely:

$$\Delta^{(\mathrm{I})}_{(\rho)\mu\nu}(q^2) = \frac{-ig_{\mu\nu}}{q^2 - \tilde{m}^2_\rho - \Pi_{\rho\omega}(q^2)},$$
$$\Delta^{(\mathrm{I})}_{(\omega)\mu\nu}(q^2) = \frac{-ig_{\mu\nu}}{q^2 - \tilde{m}^2_\omega + \Pi_{\rho\omega}(q^2)}. \qquad (6)$$

These type-I complete propagators give the physical masses of ρ^0 and ω as follows:

$$m^2_\rho = \tilde{m}^2_\rho + \Pi_{\rho\omega}(m^2_\rho), \qquad m^2_\omega = \tilde{m}^2_\omega - \Pi_{\rho\omega}(m^2_\omega). \quad (7)$$

It is well-known that the physical mass obtained from the pole of the complete propagator should be equal to the one obtained from the orthogonal rotating ρ^0 and ω to their mass eigenstates.

3. When we write the interaction Lagrangian (5), we have conveniently assumed that the widths of ρ and ω are generated dynamically by pion loops. This assumption can be dismissed, and it does not affect the following formal discussion.

In Heisenberg picture, the transition matrix (1) can be expressed in terms of LSZ reduction formula [11]

$$\langle \pi(k)\gamma(q_1), \mathrm{out}|V(q_2), \mathrm{in}\rangle = i^3 (Z_3^{(\rho)} Z_3^{(\pi)} Z_3^{(\gamma)})^{-\frac{1}{2}}$$
$$\times \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{d}^4z \frac{e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2\omega_{\vec{\mathbf{k}}}}}$$
$$\times \sum_{\lambda,\sigma} \frac{\epsilon^\mu_{\vec{\mathbf{q}}_1\lambda} e^{-iq_1\cdot y}}{\sqrt{(2\pi)^3 2\omega_{\vec{\mathbf{q}}_1}}} \frac{e^\nu_{\vec{\mathbf{q}}_2\sigma} e^{-iq_2\cdot z}}{\sqrt{(2\pi)^3 2\omega_{\vec{\mathbf{q}}_2}}}$$
$$\times (\partial^2_x + m^2_\pi)\partial^2_y(\partial^2_z + m^2_\rho)(0|T\{\pi(x)A_\mu(y)V_\nu(z)\}|0)_{\mathrm{H}}, (8)$$

where Z_3 are the renormalization constants of the wave function, and the subscript H denotes the Heisenberg picture. The above expression can be transformed into the interaction picture via

$$(0|T\{\pi(x)A_\mu(y)V_\nu(z)\}|0)_{\mathrm{H}} =$$
$$\langle 0|T\{\pi(x)A_\mu(y)V_\nu(z)e^{i\int \mathrm{d}^4x'(\mathcal{L}_{\mathrm{I}}+\mathcal{L}_{\mathrm{ChPT}}+\mathcal{L}_{\mathrm{WZW}})(x')}\}|0\rangle_{\mathrm{I}}, \qquad (9)$$

where the subscript I denotes the interaction picture.

In the language of Feymann diagrams, the non-perturbative results can be obtained by summing all diagrams of the perturbative expansion (fig. 2). Here we focus on $\rho^0 \to \pi^0\gamma$ decay, and the discussion for $\omega \to \pi^0\gamma$

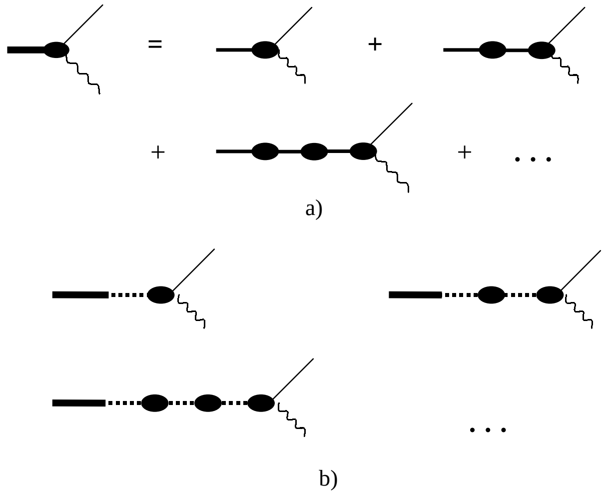

Fig. 2. Diagrams for $\rho^0 \to \pi^0\gamma$ decay. Here • denotes all potential meson loop corrections, thick solid lines and dashed lines denote type-I complete propagators of ρ^0 and ω, which are defined in fig. 1, "$\cdots$" denotes all potential high-order diagrams. In addition, the doubly thick solid lines in b) are defined as complete propagators (not type I) of ρ^0 in a). Thus the chain approximation in a) corresponds to the renormalization of the mass and wave function of the external line ρ^0, and the chain approximation in b) corresponds to the one of the internal line ω.

is similar. In fig. 2, every • denotes a contribution from all potential meson loops. Thus, the renormalization of the mass and wave function of ρ^0 and ω is necessary. It should be pointed out that every thick solid line or thick dashed line[4] in fig. 2 is defined in fig. 1. This means that the effect of ρ^0-ω mixing has been included completely via summing all diagrams of fig. 2.

There are two kinds of contributions to $\rho^0 \to \pi^0\gamma$ decay. They are shown in the fig. 2a) and b), and, respectively, correspond to the non-resonant contribution and the contribution of resonance exchange. Renormalization of mass and wave function of the external line ρ^0 is present in both fig. 2a) and 2b), while the one of the internal line ω is present in b) only. For the external boson line, Dyson has shwon in the adiabatic limit [12] that

$$(q^2 - m^2)\Delta_{F}(q^2)|_{q^2=m^2} \longrightarrow iZ_3^{\frac{1}{2}}, \qquad (10)$$

where $\Delta_F(q^2)$ denotes the chain approximation to the exact propagator, in which the mass renormalization has been performed. This relation holds for all the external ρ^0, π^0 and photon fields. Meanwhile, the renormalization of mass and wave function of the internal line ω yields its complete propagator as follow:

$$\Delta^{(\mathrm{c})}_{(\omega)\mu\nu}(q^2) = \frac{-ig_{\mu\nu}}{q^2 - m^2_\omega + i\Gamma_\omega(q^2)\sqrt{q^2}}. \qquad (11)$$

[4] We should distinguish the doubly thick solid line from the thick solid line in fig. 2. The former is defined as complete propagator of ρ^0, while the latter is defined as type-I complete propagator of ρ^0.

Here the dynamical width $\Gamma_\omega(q^2)$ is generated by pion loops. It can be also determined by the unitarity of the S matrix.

According to the above discussion, the LSZ reducation formula (8) becomes

$$\langle\pi^0(k)\gamma(q_1),\mathrm{out}|\rho^0(q_2),\mathrm{in}\rangle = if^{(c)}_{\rho^0\pi^0\gamma}(q_2^2)|_{q_2^2=m_\rho^2}\epsilon^{\mu\nu\alpha\beta}q_{1\mu}\epsilon^*_\nu q_{2\alpha}e_\beta, \tag{12}$$

where the superscript (c) denotes the non-perturbative coupling,

$$f^{(c)}_{\rho^0\pi^0\gamma}(q^2) = \bar{f}_{\rho^0\pi^0\gamma}(q^2) + \frac{\Pi_{\rho\omega}(q^2)\bar{f}^{(0)}_{\omega\pi^0\gamma}(q^2)}{q^2-m_\omega^2+i\sqrt{q^2}\Gamma_\omega(q^2)}. \tag{13}$$

In eq. (13), $\bar{f}_{\rho^0\pi^0\gamma}(q^2)$ and $\bar{f}_{\omega\pi^0\gamma}(q^2)$ denote renormalized form factors (momentum-dependent coupling), which include a meson loop correction. Labeling them by a superscript (0) and taking q^2 as mass shell of ρ, eq. (13) is just the first of equations (3). A similar discussion holds for the second of equation (3). So that we finish the prove for eq. (3) in the effective field theory formalism.

2.2 Mixed-propagator approach

Alternatively, there is another well-known quantum-mechanics method to deal with the ρ-ω mixing problem: the approach of mixed propagator [13,14]. This approach was developed even before the discovery of QCD. The vector meson propagator is given by (Renard representation)

$$D_{\mu\nu}(q^2) = \int \mathrm{d}^4x e^{-iq.x}\langle 0|T\{V_\mu(x)V_\nu(0)\}|0\rangle = D(q^2)g_{\mu\nu} + \frac{1}{q^2}(D(0)-D(s))q_\mu q_\nu\,, \tag{14}$$

where $s\equiv q^2$, and the propagator function $D(s)$ is written in the following way:

$$D(s) = \frac{1}{s-W(s)}\,. \tag{15}$$

For multi-vector-meson channels, $W(s)$ is the complex mass-square matrix with non-zero off-diagonal elements in general.

In order to define the physical states measured by experiment, let us consider the single-vector-meson resonance channel case first. The $D_{\mu\nu}$ of eq. (14) is now the ordinary propagator of a vector meson. The reaction amplitude for a process by the exchange of this vector meson resonance reads

$$\mathcal{M}\sim J_1^\mu D_{\mu\nu}J_2^\nu = (J_1\cdot J_2)\frac{1}{s-W(s)}\,, \tag{16}$$

where J_1^μ and J_2^ν represent some currents and $q_\mu J^\mu_{1,\,2}=0$. The reaction probability is

$$\sigma(s)\sim|\mathcal{M}|^2\propto\left|\frac{1}{s-W(s)}\right|^2. \tag{17}$$

The meson resonance mass measured in the experiment is real and is determined by the location of the maximum of the $\sigma(s)$ peak in the real s-axis. Then the resonance mass is determined by the following equation:

$$\frac{\partial}{\partial s}|s-W(s)|^2 = 0\,. \tag{18}$$

Consequently, the mass square of the resonance, M^2, is determined by the solution of above equation, *i.e.*,

$$s = \left[\mathrm{Re}W - \frac{1}{4}\frac{\partial}{\partial s}(W^*W+WW^*)\right] \times\left(1-\frac{\partial}{\partial s}\mathrm{Re}W\right)^{-1} \equiv M^2(s). \tag{19}$$

Now return to the ρ^0-ω two-channel case, $W(s)$ and hence $M^2(s)$ are complex and real 2×2 matrices, respectively. The mass determination equation for physical resonance states should read

$$\det[s-M^2(s)] = 0\,. \tag{20}$$

The physical states then are the eigenvectors of the real mass matrix M^2. Following refs. [13,14] and using the Breit-Wigner approximation, we have

$$W = \begin{pmatrix} m^2_{\rho_\mathrm{I}} - i\sqrt{s}\Gamma_{\rho_\mathrm{I}}\,, & \Pi_{\rho\omega}(s) \\ \Pi_{\rho\omega}(s)\,, & m^2_{\omega_\mathrm{I}} - i\sqrt{s}\Gamma_{\omega_\mathrm{I}} \end{pmatrix},$$

where m_{ρ_I} and m_{ω_I} are the masses of ρ and ω in the isospin basis. Considering $\mathrm{Im}\Pi_{\rho\omega}$ is small and hence ignorable [15], we get

$$\mathrm{Re}W = \begin{pmatrix} m^2_{\rho_\mathrm{I}}\,, & \Pi_{\rho\omega}(s) \\ \Pi_{\rho\omega}(s)\,, & m^2_{\omega_\mathrm{I}} \end{pmatrix}.$$

Then, in $\partial\mathrm{Re}W/\partial s$, only off-diagonal elements of $\partial\Pi_{\rho\omega}/\partial s$ are left. Generally, for a broad class of models $\Pi_{\rho\omega}(s)$ at (ρ,ω) resonance, the energy region can be determined by taking the VMD-type ρ-ω mixing Lagrangian $\mathcal{L}_{\rho\omega} = f_{\rho\omega}\rho^{\mu\nu}\omega_{\mu\nu}$ ($V^{\mu\nu}=\partial^\mu V^\nu-\partial^\nu V^\mu$, $V=\rho,\omega$). This leads to $\Pi_{\rho\omega}(s)=f_{\rho\omega}s$ which satisfies $\Pi(s=0)_{\rho\omega}=0$, required by the generic considerations in ref. [16]. Thus, we have

$$\frac{\partial}{\partial s}\Pi_{\rho\omega} = f_{\rho\omega} = \frac{\Pi_{\rho\omega}}{s}|_{s\sim m_\rho^2}\simeq 6.7\times10^{-3}\ll 1 \implies 1-(\partial\mathrm{Re}W/\partial s)\simeq 1, \tag{21}$$

where $|\Pi_{\rho\omega}|\simeq 4000\,\mathrm{MeV}^2$ [16] has been used in the estimation. Furthermore, noting that $\Gamma_\omega/m_\omega\ll1$, we have

$$M^2 = \begin{pmatrix} m^2_{\rho_\mathrm{I}} - \frac{\partial(s\Gamma_\rho^2)}{\partial s}, & \Pi_{\rho\omega} - \frac{1}{2}(m^2_{\rho_\mathrm{I}}+m^2_{\omega_\mathrm{I}})\frac{\partial}{\partial s}\Pi_{\rho\omega} \\ \Pi_{\rho\omega} - \frac{1}{2}(m^2_{\rho_\mathrm{I}}+m^2_{\omega_\mathrm{I}})\frac{\partial}{\partial s}\Pi_{\rho\omega}, & m^2_{\omega_\mathrm{I}} \end{pmatrix} \tag{22}$$

where the off-diagonal elements of the M^2 matrix represent the ρ-ω mixing in the isospin basis. The physical ρ

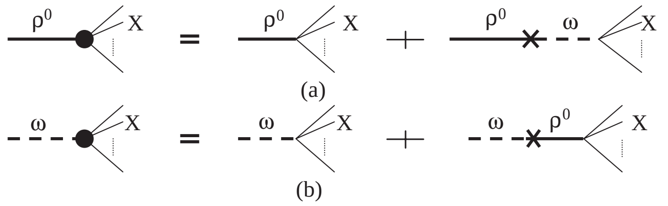

Fig. 3. The relation between the form factor of V-X vertex and the corresponding coupling constants for the vertices. The black circles in the vertex denote the form factor. The single thick solid (dashed) lines denote the ρ-propagator $D^{\mathrm{P}}_{\rho\rho}$ (ω-propagator $D^{\mathrm{P}}_{\omega\omega}$), and the thick solid cross dash (or dash cross thick solid) lines denote the mixed-propagator $D^{\mathrm{P}}_{\rho\omega}$ (or $D^{\mathrm{P}}_{\omega\rho}$). The thin lines are external lines of X-particles.

and ω are eigenstates of M^2. In other words, the M^2 matrix can be diagonalized by the unitary 2×2 matrix C:

$$CM^2C^\dagger = \begin{pmatrix} m_\rho^2\,, & 0 \\ 0\,, & m_\omega^2 \end{pmatrix},$$

where

$$C = \begin{pmatrix} 1 & -\eta \\ \eta & 1 \end{pmatrix}, \qquad \eta = -\frac{\Pi_{\rho\omega}(1-\frac{1}{2s}(m_\rho^2+m_\omega^2))}{(m_\omega^2-m_\rho^2)}\,. \quad (23)$$

Consequently, the solutions to the physical state condition of eq. (20) are as follows:

$$\begin{aligned} |\rho^0_p\rangle &= |\rho^0_{\mathrm{I}}\rangle - \eta|\omega_{\mathrm{I}}\rangle\,, \qquad \langle\rho^0_p| = |\rho^0_p\rangle^\dagger\,, \\ |\omega_p\rangle &= |\omega_{\mathrm{I}}\rangle + \eta|\rho^0_{\mathrm{I}}\rangle\,, \qquad \langle\omega_p| = |\omega_p\rangle^\dagger\,. \end{aligned} \quad (24)$$

Under this transformation, we have

$$CWC^\dagger = \begin{pmatrix} z_\rho\,, & T \\ T\,, & z_\omega \end{pmatrix}, \quad (25)$$

where $z_\rho = m_\rho^2 - im_\rho\Gamma_\rho, z_\omega = m_\omega^2 - im_\omega\Gamma_\omega$, and $T = \Pi_{\rho\omega} - \eta(z_\omega - z_\rho)$ are all defined in the physical state basis. The propagator function in the physical basis D^{P} reads

$$\begin{aligned} D^{\mathrm{P}}(s) &= C(s-W)^{-1}C^\dagger = (s-CWC^\dagger)^{-1} = \\ &\begin{pmatrix} (s-z_\rho)^{-1}\,, & (s-z_\rho)^{-1}T(s-z_\omega)^{-1} \\ (s-z_\omega)^{-1}T(s-z_\rho)^{-1}\,, & (s-z_\omega)^{-1} \end{pmatrix} \equiv \\ &\begin{pmatrix} D^{\mathrm{P}}_{\rho\rho}\,, & D^{\mathrm{P}}_{\rho\rho}TD^{\mathrm{P}}_{\omega\omega} \\ D^{\mathrm{P}}_{\omega\omega}TD^{\mathrm{P}}_{\rho\rho}\,, & D^{\mathrm{P}}_{\omega\omega} \end{pmatrix}. \end{aligned} \quad (26)$$

For the V-X vertex ($V = \rho$, ω and X represents other particles), f^{F}_{VX} denotes the corresponding form-factor, f^{P}_{VX} and f^0_{VX} denote the coupling constants in the physical basis and in the isospin basis, respectively.

Since generally $D^{\mathrm{P}}_{\rho\omega} = D^{\mathrm{P}}_{\omega\rho} \neq 0$, the form factor is different from the corresponding coupling constant, *i.e.*, $f^{\mathrm{F}}_{VX} \neq f^{\mathrm{P}}_{VX}$. From fig. 3, we have

$$D^{\mathrm{P}}_{VV}f^{\mathrm{F}}_{VX} = D^{\mathrm{P}}_{V\rho}f^{\mathrm{P}}_{\rho X} + D^{\mathrm{P}}_{V\omega}f^{\mathrm{P}}_{\omega X}, \quad (27)$$

with

$$f^{\mathrm{P}}_{\rho X} = f^{(0)}_{\rho X} - \eta f^{(0)}_{\omega X}\,, \qquad f^{\mathrm{P}}_{\omega X} = f^{(0)}_{\omega X} + \eta f^{(0)}_{\rho\gamma}. \quad (28)$$

In terms of eqs. (26) and (28) the time-like EM pion form factor is given, in the ρ-ω interference region, by

$$\begin{aligned} &F_\pi(s) = 1 + [f^{\mathrm{P}}_{\rho\gamma}D^{\mathrm{P}}_{\rho\rho}f^{F}_{\rho\pi\pi} + f^{\mathrm{P}}_{\omega\gamma}D^{\mathrm{P}}_{\omega\omega}f^{\mathrm{F}}_{\omega\pi\pi}] = \\ &1 + \frac{f^{\mathrm{P}}_{\rho\gamma}f^{\mathrm{P}}_{\rho\pi\pi}}{s-z_\rho} + \frac{f^{\mathrm{P}}_{\omega\gamma}f^{\mathrm{P}}_{\omega\pi\pi}}{s-z_\omega} + \frac{(f^{\mathrm{P}}_{\rho\gamma}f^{\mathrm{P}}_{\omega\pi\pi} + f^{\mathrm{P}}_{\omega\gamma}f^{\mathrm{P}}_{\rho\pi\pi})T}{(s-z_\rho)(s-z_\omega)} = \\ &1 + \Big(f^{\mathrm{P}}_{\rho\gamma}f^{\mathrm{P}}_{\rho\pi\pi} + \frac{(f^{\mathrm{P}}_{\rho\gamma}f^{\mathrm{P}}_{\omega\pi\pi} + f^{\mathrm{P}}_{\omega\gamma}f^{\mathrm{P}}_{\rho\pi\pi})T}{z_\rho - z_\omega}\Big) \\ &\qquad \times \Big(\frac{1}{s-z_\rho} + \xi e^{i\phi}\frac{1}{s-z_\omega}\Big), \end{aligned} \quad (29)$$

with

$$\xi e^{i\phi} = \left[\frac{1}{3}\eta - \frac{(\eta+\frac{1}{3})T}{z_\rho - z_\omega}\right]\left[1 + \frac{(\eta+\frac{1}{3})T}{z_\rho - z_\omega}\right]^{-1}, \quad (30)$$

where $f^{(0)}_{\rho\gamma} = 3f^{(0)}_{\omega\gamma}$ and $f^{(0)}_{\omega\pi\pi} = 0$ have been used, and ϕ is the Orsay phase. Using $\Pi_{\rho\omega} \simeq -4000\,\mathrm{MeV}^2$ [16] in eq. (30), we obtain that $\xi \simeq 0.012$ and ϕ is equal to about $100°$–$101°$ as s varies from m_ρ^2 to m_ω^2. These predictions are in good agreement with experimental data [17,15], and hence the mixed-propagator approach is legitimate to describe the ρ-ω mixing effects in the pion EM form factor.

Now, we study the anomalous-like $\rho^0 \to \pi^0\gamma$ and $\omega \to \pi^0\gamma$ decays in terms of the mixed-propagator approach. Namely, taking $X = \pi^0\gamma$ in eq. (27), we have

$$\begin{aligned} f^{\mathrm{F}}_{\rho^0\pi^0\gamma} &= f^{(0)}_{\rho^0\pi^0\gamma} - \eta f^{(0)}_{\omega\pi^0\gamma} + \frac{T}{m_\rho^2 - m_\omega^2 + im_\omega\Gamma_\omega}f^{(0)}_{\omega\pi^0\gamma}, \\ f^{\mathrm{F}}_{\omega\pi^0\gamma} &= f^{(0)}_{\omega\pi^0\gamma} + \eta f^{(0)}_{\rho^0\pi^0\gamma} + \frac{T}{m_\omega^2 - m_\rho^2 + im_\rho\Gamma_\rho}f^{(0)}_{\rho\pi^0\gamma}, \end{aligned} \quad (31)$$

where eqs. (24) and (26) have been used. Considering $|\eta| \simeq 0.006 \ll 1$ and $T \simeq \Pi_{\rho\omega}$, we finally obtain eq. (3) in the mixed-propagator formalism.

3 Numeric result and conclusion

The $\omega \to \pi^+\pi^-$ decay suggests that the on-shell amplitude $\Pi_{\rho\omega}(m_\rho^2)$ is around -4000 MeV^2 which is indeed very small for most isospin conservation processes. However, for $\rho^0 \to \pi^0\gamma$ we can see that $m_\rho^2 - m_\omega^2 + im_\omega\Gamma_\omega \simeq (-18624 + 6577i)$ MeV^2 is also small due to the narrow width of ω. Taking $\Pi_{\rho\omega}(m_\rho^2) \simeq -4000$ MeV^2 and combining with eq. (2), we have $f_{\rho^0\pi^0\gamma}/f^{(0)}_{\rho^0\pi^0\gamma} \simeq 1.6$ in the large-N_c limit and in the chiral limit. Therefore, the hidden isospin breaking process $\rho^0 \to \omega \to \pi^0\gamma$ indeed plays a significant role in $\rho^0 \to \pi^0\gamma$ decay. In addition, $f_{\omega\pi^0\gamma}/f^{(0)}_{\omega\pi^0\gamma} \simeq 0.99$ is obtained, so that the contribution from the ρ^0 exchange to ω decay can be omitted due to $\Gamma_\rho \gg \Gamma_\omega$ and eq. (2).

For predicting the branching ratio for ρ^0 decay precisely, the precise on-shell ρ^0-ω mixing amplitude is needed. The investigation of the ρ^0-ω mixing has been an active subject [13,17,15,10]. In ref. [15] the on-shell mixing amplitude has been determined as $\Pi_{\rho\omega}(m_\omega^2) = -[(3500 \pm 300) + (300 \pm 300)i]$ MeV^2 in the pion form factor in the time-like region, but in this fit the effect of "direct" $\omega\pi\pi$ coupling is omitted. We quote this result as R_1. Meanwhile, ref. [10] provided a complete theoretical study on this mixing, which included the effect of "direct" $\omega\pi\pi$ coupling, and is up to the next-to-leading order of the N_c^{-1} expansion. The result is $\Pi_{\rho\omega}(m_\omega^2) = -[(3956 \pm 280) + (1697 \pm 130)i]$ MeV^2, and we quote this result as R_2. Then using $B(\rho^\pm \to \pi^\pm\gamma) = (4.5 \pm 0.5) \times 10^{-4}$ and $B(\omega \to \pi^0\gamma) = (8.5 \pm 0.5) \times 10^{-2}$ and assuming $f^{(0)}_{\rho^0\pi^0\gamma} = f_{\rho^\pm\pi^\pm\gamma}$, we obtain[5]

$$B(\rho^0 \to \pi^0\gamma) = \begin{cases} (11.05 \pm 1.84) \times 10^{-4}, & \text{for } R_1, \\ (12.25 \pm 1.52) \times 10^{-4}, & \text{for } R_2. \end{cases} \tag{32}$$

Here we have used $\Pi_{\rho\omega}(m_\rho^2) \simeq m_\rho^2 \Pi_{\rho\omega}(m_\omega^2)/m_\omega^2$. The above results strongly support the fit of the solution A instead of the solution B in table 1. In fact, from the viewpoint of the experiment, solution A is also better than solution B (because of smaller χ^2 and the reasonable phase difference). Therefore, we conclude that the branching ratio for $\rho^0 \to \pi^0\gamma$ is

$$B(\rho^0 \to \pi^0\gamma) = (11.67 \pm 2.00) \times 10^{-4}. \tag{33}$$

Here we use the result of ref. [7] which was obtained from the experimental fit, instead of averaging our phenomenological estimates in eq. (32).

Finally, it is interesting to estimate the contribution of the ω exchange in $\rho^0 \to \eta\gamma$ decay. This process itself is isospin breaking due to electromagnetic interaction. The exact isospin symmetry implies $f^{(0)}_{\rho^0\eta\gamma}/f^{(0)}_{\omega\eta\gamma} = 3$. Hence a similar argument gives $f_{\rho^0\eta\gamma}/f^{(0)}_{\rho^0\eta\gamma} \simeq 1.06$ and $B(\rho^0 \to \eta\gamma)/B(\omega \to \eta\gamma) \simeq 0.5$. This result agrees with the current experimental fits, $B(\rho^0 \to \eta\gamma) = (2.4^{+0.8}_{-0.9}) \times 10^{-4}$ and $B(\omega \to \eta\gamma) = (6.5 \pm 1.0) \times 10^{-4}$ [2].

To conclude, we show that a hidden effect of isospin symmetry breaking plays an essential role in $\rho^0 \to \pi^0\gamma$ decay. The transition amplitude is derived by the effective field theory approach and the mixed-propagator approach, respectively. The results yielded by the two independent methods match each other. Our result is also supported by some experimental evidences. It indicates that the current datum for this process in PDG should be corrected.

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[5] This result includes meson loop correction, since here we use $B(\rho^\pm \to \pi^\pm\gamma)$ and $B(\omega \to \pi^0\gamma)$ to fit $f_{\rho^0\pi^0\gamma}$ and $f^{(0)}_{\omega\pi^0\gamma}$ instead of using eq. (2).